Magnetic coupling of two-dimensional pancake vortex lattices in a finite stack of thin superconducting films with transport currents in the two outermost layers

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We give a detailed study of the magnetic coupling between two-dimensional (2D) pancake vortices in a stack of N Josephson-decoupled superconducting thin films. The problem of a single pancake vortex in a finite stack of layers is first considered. We then investigate the magnetic interaction between 2D pancake lattices residing in different layers. It is assumed that all these 2D lattices have the same structure and orientation, although it is not required that they be in perfect registry. We derive an analytical solution for the coupling force on a pancake in a lattice arising from its interaction with a vortex lattice in another layer. As a direct application of this solution, we consider the case wherein a magnetic field is applied perpendicular to the layers and equal but oppositely directed surface current densities are introduced into the top and bottom layers, respectively. For weak currents, force-balanced configurations of pancake vortices are obtained. We then show the existence of a decoupling surface current density. Above this critical value, slippage occurs between 2D pancake lattices in different layers. This decoupling surface current density is then calculated for different magnetic fields and for different values of N. [S0163-1829(97)06901-4]

I. INTRODUCTION

The high-T_c cuprates have given renewed impetus in recent years to the study of layered superconductors. Because of the large anisotropy between the c axis and the CuO_2 planes in the Bi- and Tl-based compounds, it is not possible to construct a satisfactory phenomenological description using continuum Ginzburg-Landau or London formulations. Organic layered superconductors and the more recently studied epitaxially grown multilayer structures[1–8] are two other examples of systems that are best modeled by a discrete set of weakly coupled superconducting layers.

Vortices in these layered materials differ remarkably in structure from Abrikosov vortices. For instance, tilted lines of Abrikosov vortices in continuous superconductors are replaced in the layered materials by tilted stacks of intralayer two-dimensional (2D) pancake vortices joined by interlayer Josephson strings. The description of vortices in layered structures within the Lawrence-Doniach model[9–12] has been treated extensively in the literature.[13–16] The Lawrence-Doniach description has been further simplified by entirely disregarding the effects of the weak interlayer Josephson coupling. This approach has proven to be useful in studies of vortex-lattice melting at low fields.[17,18] In particular, it is shown in Ref. 18 that the characteristic shape of the low-field 3D melting line is obtained when interlayer electromagnetic interactions are taken into account and Josephson coupling entirely neglected.

Within this model, the layered superconductor is taken as an array of parallel thin films, wherein 2D pancake vortices residing in different layers can interact with each other only via magnetic coupling.[13,14,19–21] For an applied magnetic field parallel to the c direction, these vortices form straight stacks. Because currents can flow only within the superconducting layers, a nonzero component of the applied field parallel to the layers exerts zero net force on any 2D pancake vortex. This means that even in the presence of an applied field tilted relative to the c axis, the stacks of Josephson-decoupled 2D pancake vortices remain perpendicular to the layers. The only way to destroy this alignment of pancakes would be to apply transport currents to the superconducting layers. In the absence of interlayer Josephson coupling, transport current injected into a given layer remains strictly confined to that layer. A Lorentz force acts only on the pancake vortices residing in the layer where current flows. Nevertheless, pancake vortices in different layers are magnetically coupled to each other, so that any current-induced motion of pancakes in a given layer gives rise to motion of pancakes belonging to current-free layers. This is essentially the idea behind the early work on dc flux transformers[22–27] and the more recent experiments done on high-T_c superconductors.[28–33]

None of the theoretical work cited above has incorporated the effects arising from the finite number of superconducting layers present in real samples. This is regrettable, especially if one wishes to model experiments similar to those on dc flux transformers where transport currents are injected into the outermost layers of the sample. An attempt has been made in Ref. 34 to calculate the magnetic field and current distribution generated by a single 2D pancake vortex in a finite stack of Josephson-decoupled superconducting layers. This approach involves replacing all the screening layers above and below the 2D pancake layer with a superconducting continuum that allows supercurrents to flow only parallel to the layers.

We present in this paper a thorough investigation of the magnetic coupling between 2D pancake vortices in a stack of N superconducting films with no Josephson coupling, taking into account the full discreteness of the layered structure. We give an analytical solution for the magnetic force exerted by a 2D vortex lattice on a pancake vortex belonging to another lattice in a different layer. It is assumed that these two lat-

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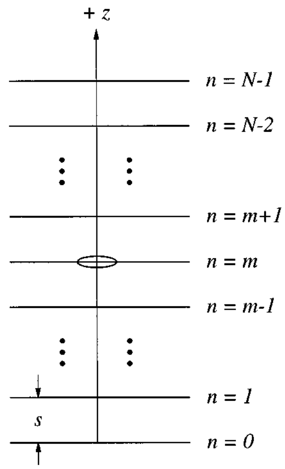

FIG. 1. A single 2D pancake vortex in a finite stack of N superconducting layers. Zero Josephson coupling between the layers is assumed and no thermal fluctuations or pinning within the layers is taken into account. The interlayer spacing is s, and each layer is referenced via an integer index n ranging from 0 to $N-1$. The z axis is perpendicular to the layers, such that $z=ns$. Hence, the plane $z=0$ coincides with the bottom layer. Lastly, $n=m$ corresponds to the layer containing the 2D pancake vortex.

tices have the same structure and are displaced (not rotated) relative to each other. As an application, we study the response of N perfectly triangular 2D pancake lattices (one in each layer) to two equal but oppositely directed current distributions flowing in the top and bottom layers. For simplicity, we assume that these currents flow perpendicular to a nearest-neighbor direction. The spatial configuration of the 2D pancake vortices is determined for sufficiently weak applied currents. We also demonstrate the existence of a maximum surface current density; above this value, slippage occurs between pancake lattices in different layers. Lastly, we determine the magnitude of this decoupling current density for different values of N and for different magnetic fields applied along the c axis.

II. SINGLE PANCAKE IN A FINITE STACK OF THIN SUPERCONDUCTING LAYERS

We begin by calculating the magnetic vector potential for a 2D pancake vortex in a stack of N superconducting layers. It is assumed throughout that no Josephson coupling is present between the layers. We also emphasize that the effects of thermal fluctuations or pinning within the layers are not taken into account.

Defining s to be the interlayer spacing, we consider each of the N layers as a thin film whose thickness d is much less than the bulk penetration depth λ_s. Assume that the bottom layer coincides with the plane $z=0$. Hence, we can reference the layer at $z=ns$ by the integer n (with $0\leqslant n\leqslant N-1$.) Let layer m correspond to the pancake layer. For the time being, choose the pancake to be centered on the z axis. (Refer to Fig. 1.)

We adopt an approach similar to that in Ref. 20. In cylindrical coordinates, the vector potential has only one nonzero component,

$$a_\varphi(\rho,z,m)=\int_0^\infty dqA(q,m)J_1(q\rho)Z(q,z,m), \qquad (1)$$

where $J_1(q\rho)$ is the first-order Bessel function of the first kind. $Z(q,z,m)$ has the following forms: for $n\leqslant z/s\leqslant n+1$, $0\leqslant n\leqslant N-1$,

$$Z(q,z,m)=\frac{1}{\sinh qs}\{e^{-Q_{n,m}|n-m|s}\sinh q[(n+1)s-z] + e^{-Q_{n+1,m}|n+1-m|s}\sinh q(z-ns)\}; \qquad (2)$$

for $z/s=n$, $0\leqslant n\leqslant N-1$,

$$Z(q,z,m)=e^{-Q_{n,m}|n-m|s}; \qquad (3)$$

otherwise,

$$Z(q,z,m) =\begin{cases} e^{-Q_{N-1,m}(N-1-m)s}e^{-q[z-(N-1)s]}, & z/s>N-1 \\ e^{-Q_{0,m}ms}e^{qz}, & z/s<0. \end{cases} \qquad (4)$$

$A(q,m)$ and the exponential terms $e^{-Q_{n,m}|n-m|s}$ given above are solved subject to two conditions.

The first condition is the equation relating the vector potential a_φ and the sheet current density K_φ generated by the single pancake: for $0\leqslant n\leqslant N-1$,

$$K_\varphi(\rho,n,m)=-\frac{c}{2\pi\Lambda}\left[a_\varphi(\rho,z,m)\Big|_{z=ns}-\frac{\phi_o}{2\pi\rho}\delta_{n,m}\right]. \qquad (5)$$

The 2D thin-film screening length Λ is here defined as $2\lambda_s^2/d$. (As mentioned in Ref. 20, some papers in the literature define Λ without the factor of 2.) A symbol such as ∂_φ denotes a partial derivative with respect to the subscripted coordinate. For the phase γ of the order parameter, we have used the gauge $\gamma=-\varphi$ for $n=m$ and $\gamma=0$ otherwise. ϕ_o is the flux quantum $hc/2e$ and $\delta_{n,m}$ is the usual Kronecker delta.

The second condition gives K_φ in terms of the discontinuity in the radial component of the magnetic field across a layer. In terms of the vector potential a_φ, we can write this condition as

$$K_\varphi(\rho,n,m)=-\frac{c}{4\pi}[\partial_z a_\varphi(\rho,z,m)|_{z=ns^+} - \partial_z a_\varphi(\rho,z,m)|_{z=ns^-}], \qquad (6)$$

for $0\leqslant n\leqslant N-1$. Let us combine this with the first condition by eliminating K_φ. We arrive at the following condition for the vector potential at any layer $0\leqslant n\leqslant N-1$:

$$a_\varphi(\rho,z,m)|_{z=ns}=\frac{\Lambda}{2}[\partial_z a_\varphi(\rho,z,m)|_{z=ns^+} - \partial_z a_\varphi(\rho,z,m)|_{z=ns^-}]+\frac{\phi_o}{2\pi\rho}\delta_{n,m}. \qquad (7)$$

We can substitute our expression for the vector potential into the above condition. Taking the Hankel transform of both sides of the resulting equation, we arrive at solutions for $A(q,m)$ and the quantities $e^{-Q_{n,m}|n-m|s}$.

In order to write the aforesaid solutions explicitly, we introduce two functions:

$$f(q)=(1+2/q\Lambda)\sinh qs+\cosh qs, \tag{8}$$

$$g(q)=2\,[(1/q\Lambda)\sinh qs+\cosh qs]. \tag{9}$$

From these, we can construct the following set:

$$h(q,n\geqslant 0)=\begin{cases} e^{-qs}, & n=0,\\ 1/f(q), & n=1,\\ 1/[g(q)-h(q,n-1)], & n>1. \end{cases} \tag{10}$$

In terms of the above quantities,

$$e^{-Q_{n,m}|n-m|s}$$
$$=\begin{cases} \prod_{p=0}^{m-n-1} h(q,m-p), & 0\leqslant n\leqslant m-1,\\ 1, & n=m,\\ \prod_{p=0}^{n-m-1} h(q,N-1-m-p), & m+1\leqslant n\leqslant N-1, \end{cases} \tag{11}$$

and

$$A(q,m)=\frac{\phi_o\sinh qs}{\pi q\Lambda[g(q)-h(q,N-1-m)-h(q,m)]}. \tag{12}$$

Aside from s and Λ, there is another important length in the problem. This is the effective penetration depth $\lambda_\parallel$ for the decay of fields induced by currents flowing parallel to the layers. This is defined by $\lambda_\parallel=s\lambda_s^2/d$. It follows from this definition that[20]

$$\Lambda=2\lambda_\parallel^2/s. \tag{13}$$

We assign values to s and $\lambda_\parallel$ that are characteristic of one of the high-temperature superconductors. (The validity of the above results are, nevertheless, not confined to such values.) In Bi-2212, we typically have $s\approx 15$ Å and $\lambda_\parallel\approx 2500$ Å (which is about $167s$). These put the value of Λ at approximately 8.3×10^5 Å or $5.6\times 10^4 s$ at temperatures close to $T=0$.

Although the model described above puts no restriction on the number of layers that can be considered, we shall limit our present investigation to stacks with thicknesses of about $\lambda_\parallel$ or less. The thicknesses of high-temperature superconducting thin films are often less than 200 layers.

III. COUPLING FORCE BETWEEN 2D PANCAKE LATTICES

Assume that a magnetic field applied perpendicular to the layers gives rise to a 2D pancake lattice in each layer. At equilibrium, all these lattices are in perfect registry along the z direction (i.e., the pancakes form vertical stacks.) The average flux density B is equal to ϕ_o/A, where A is the area of a lattice unit cell.

Let us now pick two such lattices, one in layer i and the other in layer $j\neq i$. From equilibrium, suppose that the lattice in layer j is then displaced without any rotation relative to the lattice in layer i. Denote this displacement by $\mathbf{d}_j$. What is the coupling force between two such pancake lattices?

To answer the above question, let us first focus our attention on one pancake from each of the two layers. Choose the z axis to be the axis of the pancake in layer i so that the other pancake has coordinates $\mathbf{r}_j=(\rho_j,\varphi_j)$. Let $\mathbf{K}(\mathbf{r}_j,j,i)$ be the sheet current density generated by the pancake in layer j at the position of the pancake in layer i. The force exerted on the latter by the former pancake is

$$\mathbf{F}(\mathbf{r}_j,j,i)=\mathbf{K}(\mathbf{r}_j,j,i)\times(\phi_o/c)\hat{\mathbf{z}} \tag{14}$$

for a magnetic field in the $+z$ direction. Using the results obtained in the previous section, we find that the above equation can be written as

$$\mathbf{F}(\mathbf{r}_j,j,i)=\hat{\rho}(\varphi_j)\frac{\phi_o^2}{2\pi^2\Lambda^2}\int_0^\infty dq\,\frac{J_1(q\rho_j)}{q}C(q,j,i), \tag{15}$$

where $\hat{\rho}(\varphi_j)=\hat{i}\cos\varphi_j+\hat{j}\sin\varphi_j$ and

$$C(q,j,i)=\frac{\sinh qs\;Z(q,i,j)}{g(q)-h(q,N-1-j)-h(q,j)}. \tag{16}$$

Note that $Z(q,i,j)=e^{-Q_{i,j}|i-j|s}$ in Eq. (16). $\mathbf{F}(\mathbf{r}_j,j,i)$ is an attractive force that tends to bring the two pancake vortices into alignment. This can be shown explicitly when $\rho\ll\lambda_\parallel$ and $0<|i-j|s\ll\lambda_\parallel$, for which Eq. (15) reduces to

$$\mathbf{F}(\mathbf{r}_j,j,i)\approx\hat{\rho}(\varphi_j)\left(\frac{\phi_o}{2\pi\Lambda}\right)^2\frac{\sqrt{\rho_j^2+|i-j|^2s^2}-|i-j|s}{\rho_j}, \tag{17}$$

with the unit vector $\hat{\rho}(\varphi_j)$ pointing from the z axis (passing through the center of pancake i) to the vertical axis through the center of pancake j.

Analogous to the approach taken in Refs. 25–27, we then write the coupling force on a pancake in layer i due to its interaction with the vortex lattice in layer j as a sum over reciprocal lattice vectors $\mathbf{g}$:

$$\mathbf{F}_c(\mathbf{d}_j,j,i)=\frac{1}{A}\sum_{\mathbf{g}\neq 0}\mathbf{G}(\mathbf{g},j,i)e^{i\mathbf{g}\cdot\mathbf{d}_j}. \tag{18}$$

The vector $\mathbf{G}(\mathbf{g},j,i)$ in the above equation is the 2D Fourier transform of $\mathbf{F}(\mathbf{r},j,i)$:

$$\mathbf{G}(\mathbf{g},j,i)=\int d^2r e^{-i\mathbf{g}\cdot\mathbf{r}}\,\mathbf{F}(\mathbf{r},j,i). \tag{19}$$

We can evaluate $\mathbf{G}$ explicitly with the aid of Eq. (15). This evaluation yields

$$\mathbf{G}(\mathbf{g},j,i)=-\frac{i\phi_o^2}{\pi\Lambda^2}\frac{\mathbf{g}}{g^3}C(g,j,i), \tag{20}$$

with g denoting the magnitude of $\mathbf{g}$. Observe that $\mathbf{G}(-\mathbf{g},j,i)=-\mathbf{G}(\mathbf{g},j,i)$. Therefore,

$$\mathbf{F}_c(\mathbf{d}_j,j,i)=\frac{\phi_o^2}{\pi\Lambda^2A^2}\sum_{\mathbf{g}\neq 0}\frac{\mathbf{g}}{g^3}C(g,j,i)\sin(\mathbf{g}\cdot\mathbf{d}_j). \quad (21)$$

The above result is particularly convenient for the case of high magnetic fields (i.e., fields in the order of a tesla.) For this case, only a few reciprocal lattice vectors are needed to obtain a good approximate value for the coupling force.

The opposite limit is the case of very weak magnetic fields and displacements d_j that are much smaller than the nearest-neighbor distance of the pancake lattices. One can show from the above results that

$$\lim_{B\to 0}\mathbf{F}_c(\mathbf{d}_j,j,i)=\mathbf{F}(\mathbf{d}_j,j,i). \quad (22)$$

The coupling force thereby reduces to a pairwise interaction between pancake vortices.

IV. TRANSPORT CURRENTS AT THE TOP AND BOTTOM LAYERS

Consider the special case when the 2D pancake lattices discussed in the previous section are perfect triangular lattices. Let a denote the nearest-neighbor distance in each lattice. The area A of a unit-cell parallelogram is therefore equal to $a^2\sqrt{3}/2$. At equilibrium with a perpendicular magnetic field, we had noted that the pancake lattices in the different layers will all be in perfect registry with one another.

But suppose we apply equal but oppositely directed sheet currents to the top and bottom layers. Lorentz forces arising from these currents will then move the top and bottom pancake lattices from their equilibrium positions. We shall only consider currents that preserve the original orientations of both top and bottom lattices. Such displacements will then cause the other lattices in the interior layers to move, keeping their orientations unchanged. If the magnitude of the applied currents is not too large, then all the lattices in the different layers will inevitably stop moving as they relax to new positions. In these new locations, the net force on any of the lattices will once again be zero.

For convenience, let us restrict the present discussion to an odd number of layers. Let M be a positive integer, such that the total number N of superconducting layers is equal to $2M+1$.

Let us also restrict our attention to a set of N pancakes whose equilibrium positions are aligned along the direction perpendicular to the layers. Choose the axis of this stack of pancakes as the z axis. As before, we take the bottom layer to be the plane $z=0$.

For simplicity, assume that all lattice displacements occur only along a certain nearest-neighbor direction. Choose the x axis to be along this direction. This corresponds to making the transport currents at the top and bottom layers flow parallel to the y axis. Refer to the y components of the surface current densities in the top and bottom layers as K_y^{top} and K_y^{bot} respectively, such that $K_y^{\text{top}}=-K_y^{\text{bot}}$. (See Fig. 2.)

In order to compute the final displacements of the pancakes described above, it is convenient to begin by displacing the top and bottom lattices to new positions x_{N-1} and x_0, respectively, with $x_{N-1}=-x_0$. Keeping these lattices

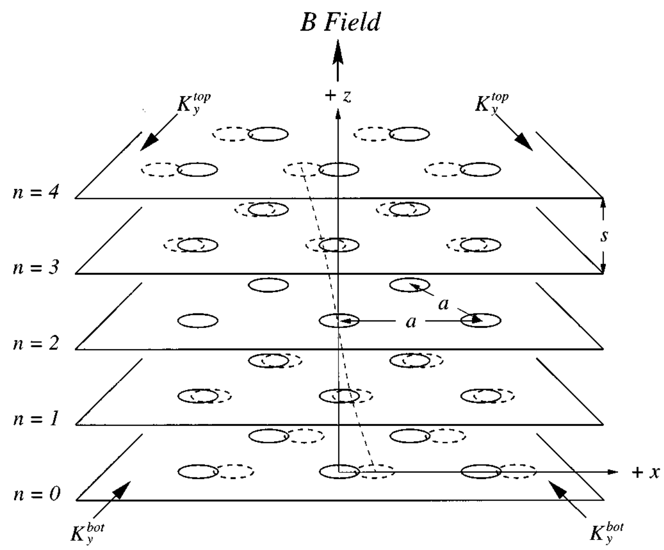

FIG. 2. 2D triangular pancake lattices in a finite stack of thin superconducting films with equal but oppositely directed transport currents flowing in the top and bottom layers. For simplicity, only the case of five layers is illustrated. The 2D pancake lattices in the different layers are assumed to interact with each other only via magnetic coupling. The y components of the surface current densities in the top and bottom layers are K_y^{top} and K_y^{bot}, respectively, with $K_y^{\text{top}}=-K_y^{\text{bot}}$. Pancakes drawn in solid lines correspond to the case when no sheet currents are applied. The pancakes outlined in dashed lines correspond to a force-balanced configuration associated with nonzero K_y^{top} and K_y^{bot}. By symmetry, the pancakes in the central layer $n=2$ are unaffected by equal but oppositely directed surface currents in the top and bottom layers. The z axis goes through five pancakes whose equilibrium positions lie along a common vertical. A sketch of these same five pancakes is shown with the top and bottom surface currents turned on (their resulting force-balanced positions connected by dashed lines).

fixed to their new positions, we then allow the lattices in the interior layers to relax until the full force-balanced configuration of pancakes is established.

Let x_i be the x coordinate of the pancake in layer i. If we neglect terms involving the 2D pancake mass, we arrive at the following force balance equations for pancake lattices in the interior layers: for $1\leqslant i\leqslant N-2$,

$$\sum_{j\neq i} F_{cx}(x_j-x_i,j,i)=\eta\dot{x}_i. \quad (23)$$

Summation symbols like $\Sigma_{j\neq i}$ indicate that we exclude $j=i$ from the summation range $0\leqslant j\leqslant N-1$. F_{cx} is the component of $\mathbf{F}_c$ along the nearest-neighbor axis and η is the viscous drag coefficient. Finally, $\dot{x}_i$ is the time derivative of x_i.

As the system approaches its final force-balanced configuration, all lattice motions in the interior layers cease:

$$\lim_{t\to\infty}\dot{x}_i=0 \quad (24)$$

for $1\leqslant i\leqslant N-2$. To obtain the final displacements of the $N-2$ pancake lattices in the interior layers, we must therefore solve the $N-2$ equations in Eq. (23) self-consistently, subject to the $N-2$ boundary conditions in Eq. (24). Actually, we can reduce this problem to $M-1$ equations in $M-1$ unknowns since, by symmetry, $x_M=0$ for all time and $x_{2M-i}=-x_i$ for $0\leqslant i\leqslant M-1$.

The equations that determine the force balance on the top and bottom vortex lattices are as follows: for the bottom layer, $i=0$,

$$\sum_{j\neq 0} F_{cx}(x_j-x_0,j,0)=-\frac{\phi_o}{c}K_y^{\mathrm{bot}}; \tag{25}$$

for the top layer, $i=N-1$,

$$\sum_{j\neq N-1} F_{cx}(x_j-x_{N-1},j,N-1)=-\frac{\phi_o}{c}K_y^{\mathrm{top}}. \tag{26}$$

In other words, for the top and bottom pancake lattices, the Lorentz force produced by either K_y^{top} or K_y^{bot} is exactly balanced by the coupling forces due to the pancake lattices in the other layers. Having self-consistently calculated the final displacements of the interior pancake lattices, we can then use either Eq. (25) or Eq. (26) to solve for the applied surface current densities flowing in the outermost layers. (Recall that $K_y^{\mathrm{top}}=-K_y^{\mathrm{bot}}$.)

The full numerical solution to the problem outlined above and other related results are discussed in the next section.

V. NUMERICAL RESULTS AND DISCUSSION

In many of the succeeding figures, we express surface current densities in units of $c\phi_o/\Lambda^2$. For $s\approx 15$ Å and $\Lambda\approx 5.6\times 10^4 s$, this unit is approximately 30 mA/cm.

Figure 3 shows plots of K_y^{bot}, the y component of the surface current density in the bottom layer, as a function of the bottom lattice displacement x_0 relative to equilibrium. The magnetic field perpendicular to the layers is 0.1 T . This is equivalent to having a pancake lattice spacing a of about $103s$. The range of values for x_0 spans half the lattice spacing. The curves can be extended antisymmetrically about $x_0=a/2$ to cover the full periodic interval $[0,\ a]$. Plots corresponding to 3, 5, 11, 51, 101, and 151 layers are given. Recall that the value of $\lambda_\parallel$ is taken to be about $167s$ for Bi-2212; hence, the values of N considered range from around 2–91 % percent of the value of $\lambda_\parallel$.

When the applied currents are small, the lattices are able to rearrange themselves so that the forces on the top and bottom pancakes arising from these currents are cancelled by the forces exerted by the lattices in the interior. But as every curve in Fig. 3 clearly shows, there is a maximum surface current density, the decoupling surface current density K_d, above which the interior pancakes are unable to generate forces on the top and bottom lattices that cancel the forces due to the applied currents. The sections of the curves for which K_y^{bot} versus x_0 have negative slopes (dashed) correspond to unstable force-balanced configurations of pancake lattices.

Figure 4 shows a few stable configurations for the $N=101$ case in Fig. 3. Notice that the scales used for the x and z axes are different. For brevity, only one pancake from each layer is shown; these pancakes would form a straight stack along the z direction in the absence of the applied currents. The stack with the largest bottom lattice displacement along x approximately corresponds to the configuration at maximum surface current density for 101 layers in a 0.1 T perpendicular field.

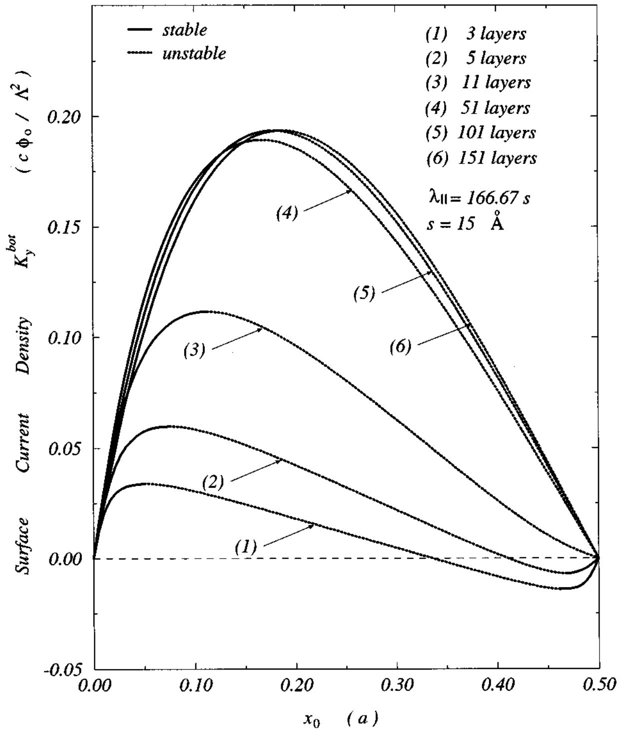

FIG. 3. Plots of the bottom layer surface current density K_y^{bot} versus the bottom lattice displacement x_0 for 3, 5, 11, 51, 101, and 151 layers. A 0.1 T magnetic field is applied perpendicular to the layers, such that $a=103.02s$.

What is immediately striking about Fig. 4 is how far the top and bottom lattices are displaced, compared with the relatively small displacements of the interior lattices. If the x and z axes were identically scaled, the pancakes in the interior layers would appear almost vertically aligned, while the top and bottom pancakes would look quite dissociated from the rest of the stack. Note, however, that the top and bottom pancakes are not really dissociated since they are still magnetically coupled to the pancakes in the interior.

Figure 5 shows semilog plots of the decoupling surface current density K_d as a function of the magnitude of the magnetic field applied perpendicular to 101 superconducting layers. The range of field values represented is between 0 T and 1.0 T. For greater detail, we have included an expanded view of that portion of the curve close to the zero-field limit.

The decoupling surface current density K_d decreases monotonically with increasing field throughout the entire range represented. Whereas the current density at 0.001 T is a significant 77% of the zero-field value, the current density at 0.1 T is only around 15%. At 1.0 T, the decoupling current density is reduced to 4% of its value at zero field.

Finally, Fig. 6 gives plots of the decoupling surface current density as a function of Ns. Curves for $B=0$ T, 0.01 T, and 0.10 T perpendicular fields are shown. The last two fields correspond to values for a of approximately $326s$ and $103s$, respectively.

As illustrated quite clearly by the three curves in Fig. 6,

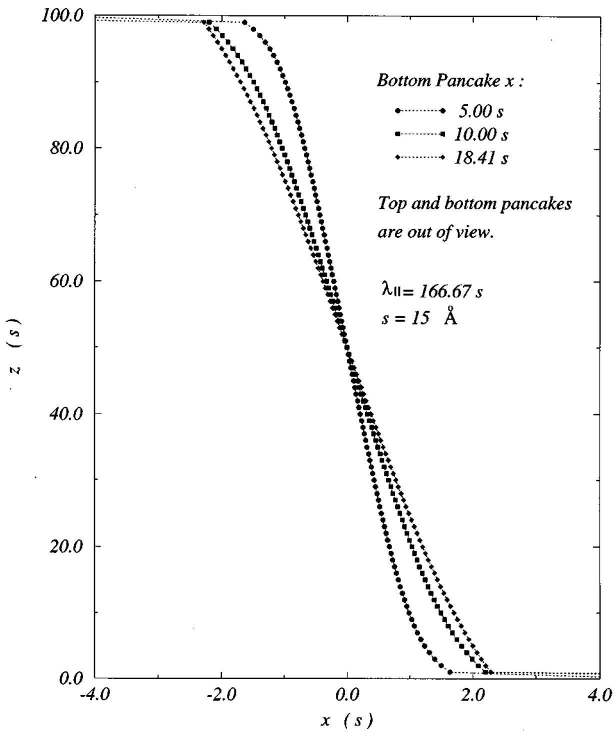

FIG. 4. Force-balanced configurations of 101 pancakes (each belonging to a 2D triangular lattice in each layer) in the presence of surface current densities K_y^{top} and K_y^{bot} flowing in the top and bottom layers, respectively. We assume that $K_y^{\mathrm{top}}=-K_y^{\mathrm{bot}}$. A 0.1 T magnetic field is applied perpendicular to the layers, such that $a=103.02s$. The magnitudes of the top and bottom surface current densities considered correspond to bottom lattice displacements of $5.0s$, $10.0s$, and $18.41s$ from equilibrium (i.e., a straight stack). The x axis is expanded relative to the z axis to show greater detail.

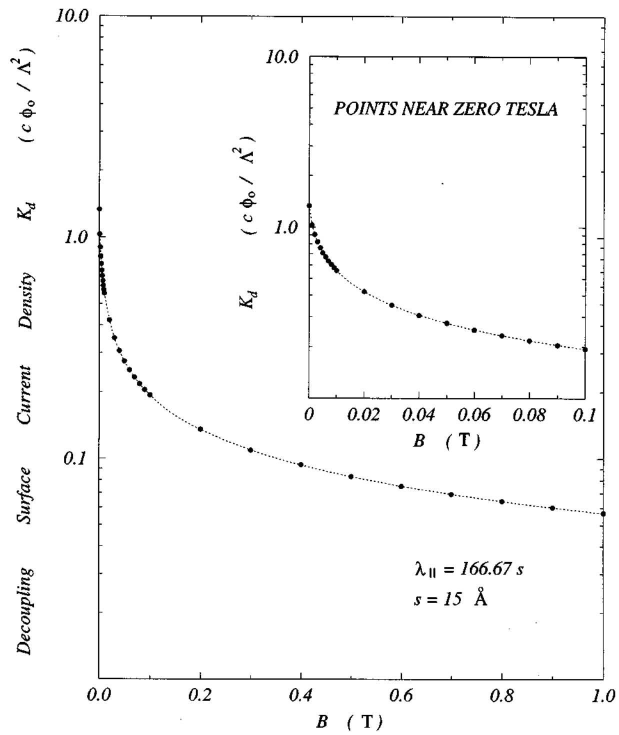

FIG. 5. Semilog plots for the magnetic field dependence of the decoupling surface current density K_d (applied in opposite directions to the top and bottom layers) needed to magnetically decouple the outermost 2D pancake lattices from those in the interior. The number of layers is 101. The value of K_d in the zero-field limit is included.

the decoupling surface current density K_d for a given perpendicular field B approaches a saturation value as the number of layers is increased. Moreover, this saturation current density increases with decreasing field before it finally approaches a constant value in the zero-field limit.

For what range of Ns values is saturation achieved for a given perpendicular magnetic field? For the zero-field curve $(a\rightarrow\infty)$ in Fig. 6, we see that this saturation is reached for $Ns\gtrsim O(\lambda_\parallel)$. This can be understood as follows. In Ref. 20, it was shown that a pancake in one layer of an infinite stack is effectively screened by the neighboring layers within a distance of $\lambda_\parallel$. Now, consider a finite stack of pancakes, one to each superconducting layer. If the number of layers exceeds $2\lambda_\parallel/s$, we expect that the pancakes deep in the interior will have no interaction with the top and bottom pancakes. The top or bottom pancake cannot couple to an interior pancake corresponding to a certain layer if that layer is at a distance greater than $\lambda_\parallel$.

The situation described above is certainly valid for low perpendicular magnetic fields. But what happens at high enough fields, such that the 2D lattice constant a is smaller than $\lambda_\parallel$? For the case of $a\rightarrow\infty$, each layer has at most one pancake that can interact with a top or bottom pancake; but if $a<\lambda_\parallel$, this is no longer true.

As an illustration, consider a pancake belonging to the lattice in the top layer. If a is significantly smaller than $\lambda_\parallel$, a sizable number of pancakes in the layer directly below the uppermost lattice will be within a distance $\lambda_\parallel$ from the top pancake. This is also true for a successively smaller number of interior pancakes as one continues on to the layers further down the stack.

Thus, we would expect the strongest coupling with a pancake in the uppermost lattice to come from pancakes belonging to layers a distance less than $\lambda_\parallel$ from the top. But how much less than $\lambda_\parallel$? Although the situation is quite complicated, the two curves in Fig. 5 for the case of $a<\lambda_\parallel$ show that this distance is $O(a)$.

Order of magnitude estimates can be obtained analytically if, to lowest order, we ignore the effects arising from the finite thickness of the superconducting stack. The case of infinite thickness has been worked out in Ref. 20; suppose that we use the results obtained from this calculation to approximate the interaction between 2D pancake lattices for a finite stack of superconducting layers. To further simplify matters, we assume that all the interior pancake lattices are in perfect registry for all values of top and bottom lattice displacements. From what has been discussed in connection with Fig. 3, we see that the last assumption is a reasonable approximation.

Let us apply the above approximation to the zero-field limit. In this limit, we have argued that the characteristic thickness for which the value of K_d saturates is of the order of $\lambda_\parallel$. For relatively thin samples, $(N-1)s\ll\lambda_\parallel$, this approximation yields $K_d\sim(c\phi_o/4\pi^2\Lambda^2)(N-1)$, whereas for

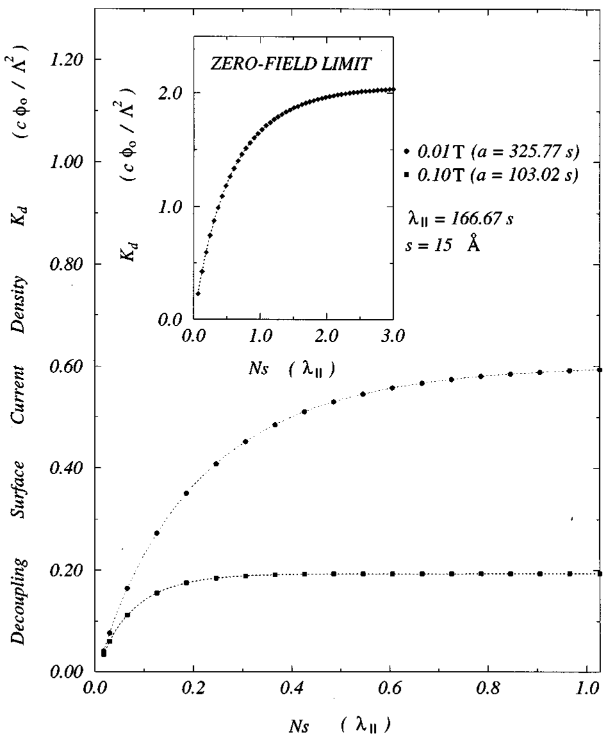

FIG. 6. Dependence on Ns of the decoupling surface current density K_d (applied in opposite directions to the top and bottom layers) needed to magnetically decouple the outermost 2D pancake lattices from those in the interior. Curves are given for $B=0$ T, 0.01 T, and 0.10 T fields applied perpendicular to the layers. The values of a corresponding to the two nonzero fields are $325.77s$ and $103.02s$, respectively.

the case of relatively thick samples, $(N-1)s \gg \lambda_{\parallel}$, it yields $K_d \sim (c\phi_o/4\pi^2\Lambda^2)(\lambda_{\parallel}/s)$. With 11 layers and $\lambda_{\parallel} \approx 167\ s$, the former estimate gives a value of about 7.5 mA/cm, which is comparable with the numerically computed value of approximately 6.9 mA/cm. The latter estimate for the same value of $\lambda_{\parallel}$ is about 126 mA/cm, which is slightly over twice the numerically calculated value of approximately 60 mA/cm.

For finite values of B, we can argue, using the same approximation, that $a\sqrt{3}/4\pi \sim 0.14a$ is the characteristic thickness for which the value of K_d saturates. Moreover, we can show that $K_d \sim (c\phi_o/4\pi^2\Lambda^2)(N-1)$ when $Ns < 0.14a$, and $K_d \sim (c\phi_o/4\pi^2\Lambda^2)(0.14a/s)$ when $Ns > 0.14a$. Assume a 0.1 T field corresponding to $a \approx 103s$. The first estimate, valid for relatively thin samples, equals approximately 1.5 mA/cm for 3 layers and $\lambda_{\parallel} \approx 167s$. This is fairly close to the numerically computed value of 1.0 mA/cm. The second estimate, involving relatively thick samples, is about 11 mA/cm for the same value of $\lambda_{\parallel}$. The corresponding value obtained numerically is approximately 6 mA/cm.

VI. SUMMARY

In this paper, we studied the magnetic coupling between 2D pancake vortices in a stack of N superconducting films. The effects of Josephson coupling, thermal fluctuations, and pinning were not taken into account. We considered a pair of 2D pancake vortex lattices residing in different layers, both having the same structure and orientation, and displaced relative to each other. An analytical expression was obtained for the coupling force on a pancake in one lattice due to its interaction with the pancakes in the other vortex lattice. Assuming perfect 2D triangular pancake lattices in each layer, we investigated the case when equal but oppositely directed surface current densities were applied to the top and bottom layers. For simplicity, the surface current directions were chosen such that all lattice displacements were along one nearest-neighbor direction. The results were as follows.

If the magnitude of the applied surface current densities was set above a certain value K_d, then the top and bottom 2D pancake lattices decoupled from the rest. Below the decoupling surface current density K_d, stacks of 2D pancake vortices formed. These stacks did not form uniformly tilted lines. Instead, the displacements of the top and bottom pancakes were shown to be large compared with those of the interior pancakes.

For fixed N, the decoupling surface current density decreased as B was increased (i.e., as the 2D pancake lattice constant a was decreased). For fixed B, the decoupling surface current density initially increased with increasing N, and then attained a saturation value. If $a < \lambda_{\parallel}$, this saturation occured when $Ns \sim O(a)$. If $a > \lambda_{\parallel}$, saturation was achieved for $Ns \sim O(\lambda_{\parallel})$.

An immediate extension of our approach is the study of the dynamics of the pancake lattices in Josephson-decoupled layers with uniform pinning, in the presence of a transport current in one of the outermost layers.[35] This extension is relevant to dc flux transformer experiments done on high-T_c superconductors, a few of which we have already mentioned in the Introduction. With our approach, theoretical V-I curves can be computed,[35] and these can then be compared with experimental data to test the validity and limitations of our model.

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